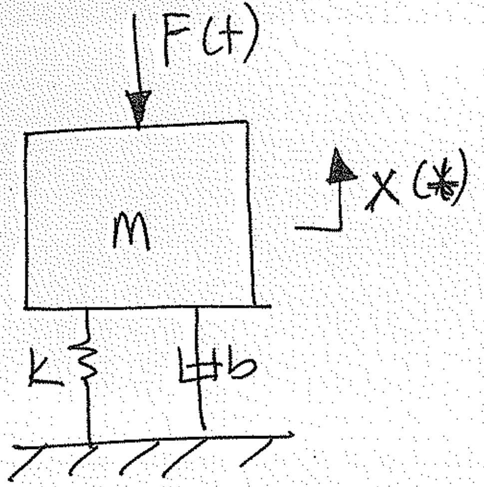
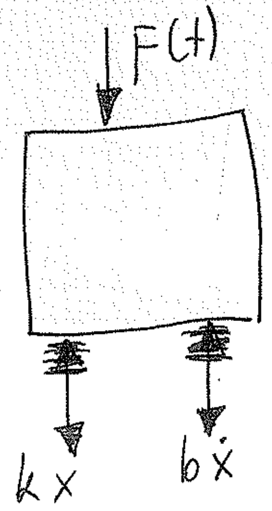


1. Newton



$\rightarrow$ yönler $x(t)$'in tersi yönünde belirlenmiştir.

Yaylar başlangıçta sıkışmış pozisyonda.

$$\sum \vec{F}_y = 0 \Rightarrow -F(t) - kx - b\dot{x} = m\ddot{x}$$

$$m\ddot{x} + b\dot{x} + kx = -F(t)$$

2. Lagrange

(Yaylar başlangıçta sıkışmış kabul edilmemiştir.)

$$K = \frac{1}{2} m \dot{x}^2$$

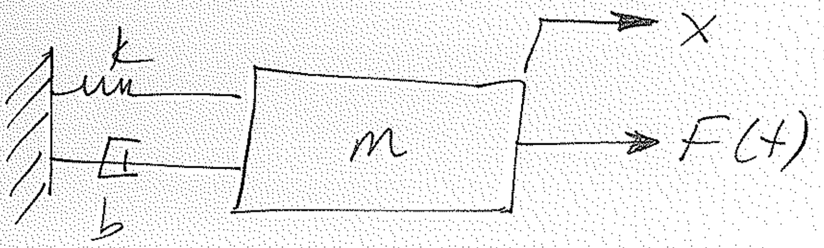
$$P = \frac{1}{2} k x^2$$

$$D = \frac{1}{2} b \dot{x}^2$$

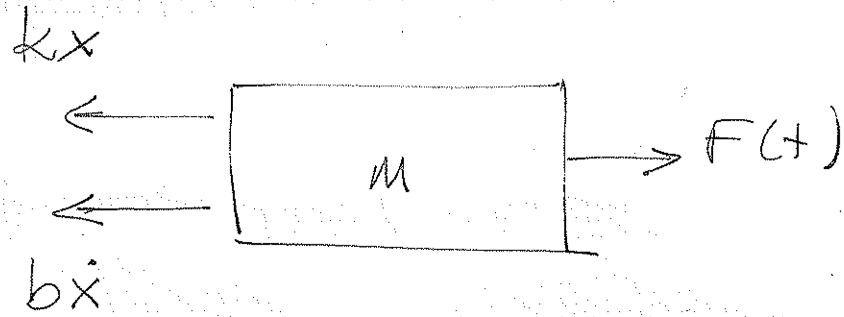
$$Q_1 = -F(t)$$

$$\Rightarrow \quad m\ddot{x} + b\dot{x} + kx = -F(t)$$

Dynamic equation of a mass-spring-damper model.



Newten Method



$$\sum \vec{F_x} = m\ddot{x} \Rightarrow -kx - b\dot{x} + F(t) = m\ddot{x}$$

$$m\ddot{x} + b\dot{x} + kx = F(t)$$

Lagranian Method

$K: \frac{1}{2} m \dot{x}^2$

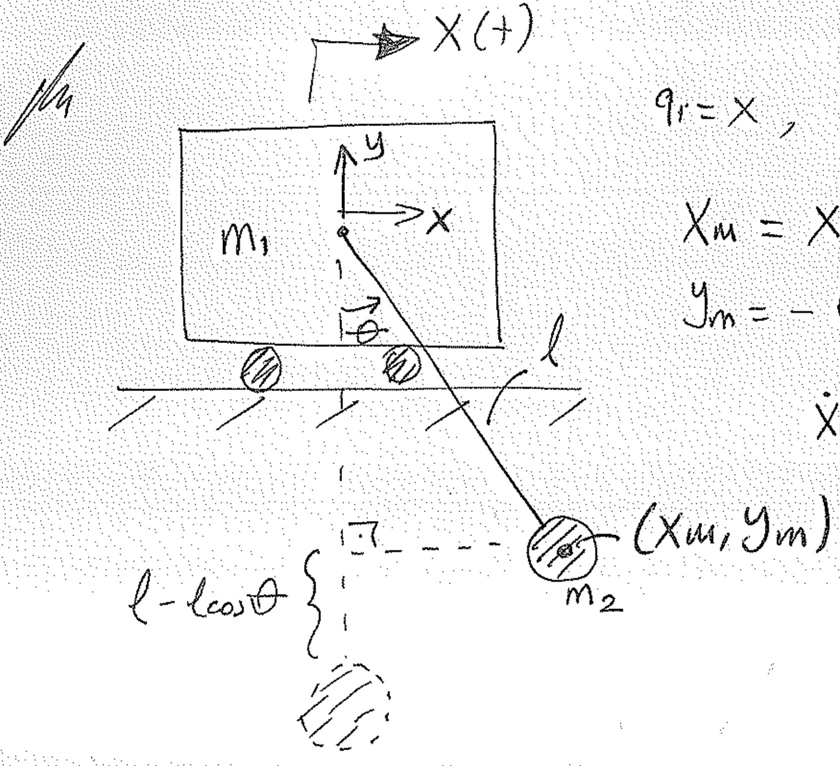
$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial K}{\partial x} + \frac{\partial D}{\partial \dot{x}} + \frac{\partial P}{\partial x} = Q_i$$

$P: \frac{1}{2} k x^2$

$$m\ddot{x} + b\dot{x} + kx = F(t)$$

$D: \frac{1}{2} b \dot{x}^2$

$Q = F(t)$



$q_1 = x, \quad q_2 = \theta$

$x_m = x + l\sin\theta$

$y_m = -l\cos\theta$

$\dot{x}_m = \dot{x} + l\dot{\theta}\cos\theta$

$\dot{y}_m = l\dot{\theta}\sin\theta$

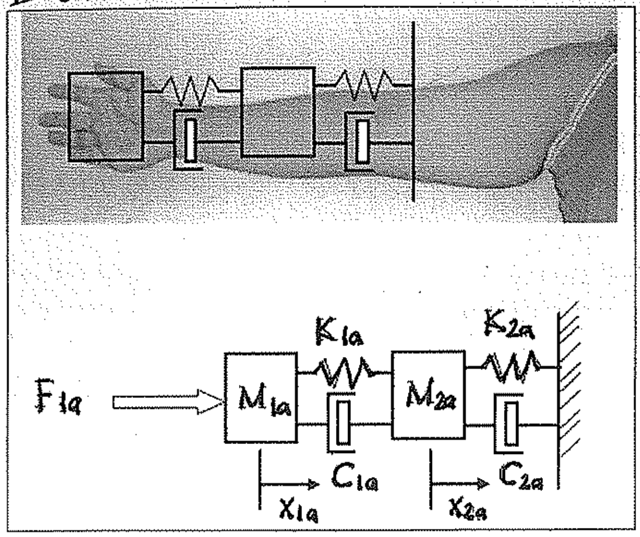
$$\Rightarrow K = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 (\dot{x}_m^2 + \dot{y}_m^2) = \frac{1}{2} m_1 \dot{x} + \frac{1}{2} m_2 (\dot{x}^2 + l^2\dot{\theta}^2\cos^2\theta + 2\dot{x}l\dot{\theta}\cos\theta + l^2\dot{\theta}^2\sin^2\theta)$$

$$K = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 (\dot{x}^2 + l^2\dot{\theta}^2 + 2\dot{x}l\dot{\theta}\cos\theta) = \frac{1}{2}(m_1 + m_2)\dot{x}^2 + \frac{1}{2} m_2 (l^2\dot{\theta}^2 + 2\dot{x}l\dot{\theta}\cos\theta)$$

$$P = m.g.l(1-\cos\theta)$$

SORU 2

~~Drive~~ the equation of motions of the following human arm model.
Derive



1. Lagrangian Yöntemi.

$$K = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

$$D = \frac{1}{2} c_1 (\dot{x}_1 - \dot{x}_2)^2 + \frac{1}{2} c_2 \dot{x}_2^2$$

$$P = \frac{1}{2} k_1 (x_1 - x_2)^2 + \frac{1}{2} k_2 x_2^2$$

I. $q_1 = x_1$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \quad , \quad \frac{\partial K}{\partial x_1} = 0 \quad , \quad \frac{\partial D}{\partial \dot{x}_1} = c_1 (\dot{x}_1 - \dot{x}_2) \quad , \quad \frac{\partial P}{\partial x_1} = k_1 (x_1 - x_2)$$

$\Rightarrow$ 1. Hareket denklemi: $m_1 \ddot{x}_1 + c_1 (\dot{x}_1 - \dot{x}_2) + k_1 (x_1 - x_2) = F_1$

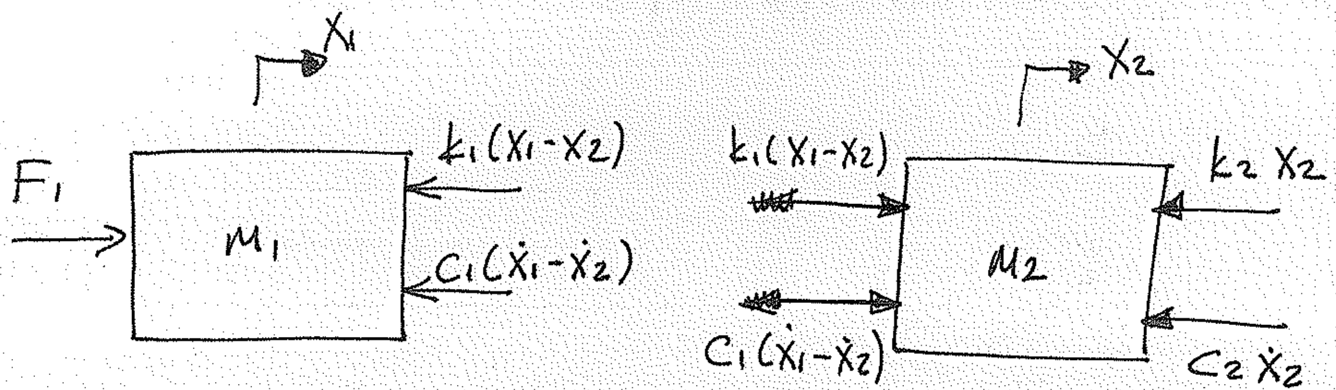
II. $q_2 = x_2$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \quad , \quad \frac{\partial K}{\partial x_2} = 0 \quad , \quad \frac{\partial D}{\partial x_2} = c_1 (\dot{x}_2 - \dot{x}_1) + c_2 \dot{x}_2$$

$$\frac{\partial P}{\partial x_2} = k_1 (x_2 - x_1) + k_2 x_2$$

$\Rightarrow$ 2. Hareket denklemi: $m_2 \ddot{x}_2 + c_1 (\dot{x}_2 - \dot{x}_1) + c_2 \dot{x}_2 + k_1 (x_2 - x_1) + k_2 x_2 = 0$

2. Newton-Euler Yöntemi kullanılırsa:



$$\sum \vec{F_x} = m_1 \vec{a_{1x}}$$

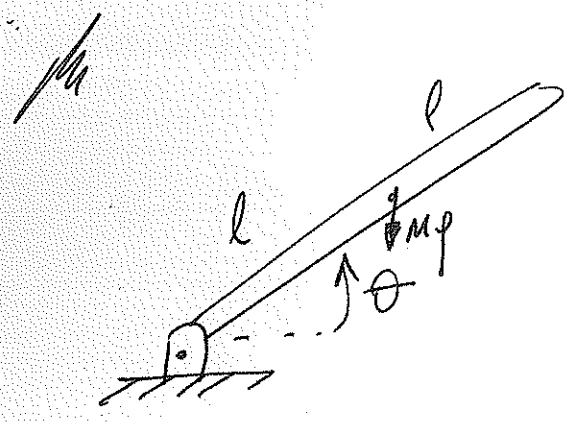
$$F_1 - k_1(x_1-x_2) - c_1(\dot{x}_1-\dot{x}_2) = m.\ddot{x}_1$$

$\Rightarrow$ 1. Hareket denk. $\quad m\ddot{x}_1 + k_1(x_1-x_2) + c_1(\dot{x}_1-\dot{x}_2) = F_1$

$$\sum \vec{F_x} = m_2 \vec{a_{2x}}$$

$$k_1(x_1-x_2) + c_1(\dot{x}_1-\dot{x}_2) - k_2 x_2 - c_2\dot{x}_2 = m_2\ddot{x}_2$$

$$\Rightarrow m_2\ddot{x}_2 + k_1(x_2-x_1) + c_1(\dot{x}_2-\dot{x}_1) + k_2 x_2 + c_2\dot{x}_2 = 0$$



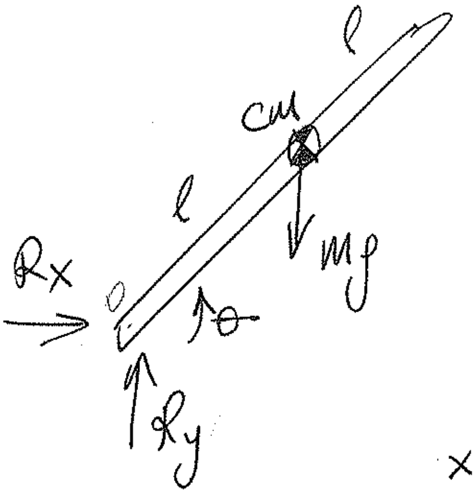
Derive the equation of motion using Newton - Euler and Lagrangian equations.

$\Sigma M_O = I_O \ddot{\theta}$

$-mg\, l\cos\theta = \frac{1}{3} m 4l^2 \ddot{\theta}$

$\Rightarrow$

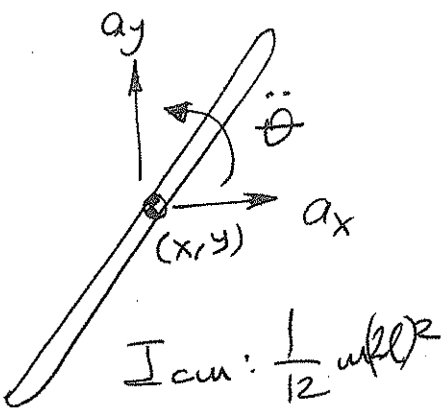
1. Newton - Euler Method.



$\Sigma \vec{F_x} = m.\vec{a_x}$

$\Sigma \vec{F_y} = m.\vec{a_y}$

$\Sigma \vec{M_{cm}} = I_{cm}.\vec{\ddot{\theta}}$



$I_{cm}: \frac{1}{12} m(2l)^2$

$x = l\cos\theta \rightarrow \dot{x} = -l\dot{\theta}\sin\theta \rightarrow \ddot{x} = -l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta$

$y = l\sin\theta \rightarrow \dot{y} = l\dot{\theta}\cos\theta \rightarrow \ddot{y} = l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta$

$R_x = m.a_x \Rightarrow -ml\ddot{\theta}\sin\theta - ml\dot{\theta}^2\cos\theta$

$R_y - mg = ma_y \Rightarrow R_y = mg + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta$

$+R_x\, l\sin\theta - R_y\, l\cos\theta = I_{cm}.\ddot{\theta}$

$(-ml\ddot{\theta}\sin\theta - ml\dot{\theta}^2\cos\theta)\, l\sin\theta - (mg + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta) *$

$* \; l\cos\theta = \frac{1}{12} m 4 l^2 \ddot{\theta}$

$\Rightarrow -ml^2\sin^2\theta\ddot{\theta} - ml^2\cancel{\dot{\theta}^2\cos\theta\sin\theta} - mg\, l\cos\theta - ml^2\ddot{\theta}\cos^2\theta$

$+ ml^2\cancel{\dot{\theta}^2\sin\theta\cos\theta} = \frac{1}{3} m l^2 \ddot{\theta}$

$-ml^2\ddot{\theta} - mg\, l\cos\theta = \frac{1}{3} ml^2\ddot{\theta} \Rightarrow \underline{\frac{4}{3} ml^2\ddot{\theta} + mg\, l\cos\theta = 0}$

2. Lagranien Yöntemi

$$K = \frac{1}{2} m \left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2} I_{cm} \cdot \dot{\theta}^2$$

$$P = \cancel{\frac{1}{2}}\, m \cdot g \cdot l \sin\theta$$

$$K = \frac{1}{2} m \left(l^2 \dot{\theta}^2 \sin^2\theta + l^2 \dot{\theta}^2 \cos^2\theta\right) + \frac{1}{2} \cdot \frac{1}{12} m 4 l^2 \dot{\theta}^2$$

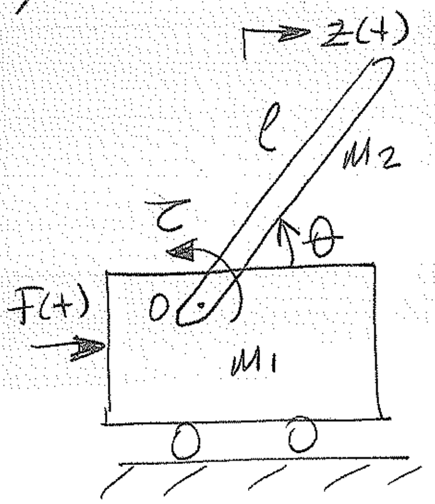
$$K = \frac{1}{2} m l^2 \dot{\theta}^2 + \frac{1}{6} m l^2 \dot{\theta}^2 = \frac{2}{3} m l^2 \dot{\theta}^2$$

$$\frac{\partial K}{\partial \dot{\theta}} = \frac{4}{3} m l^2 \dot{\theta} \Rightarrow \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}}\right) = \frac{4}{3} m l^2 \ddot{\theta}$$

$$\frac{\partial P}{\partial \theta} = mgl\cos\theta$$

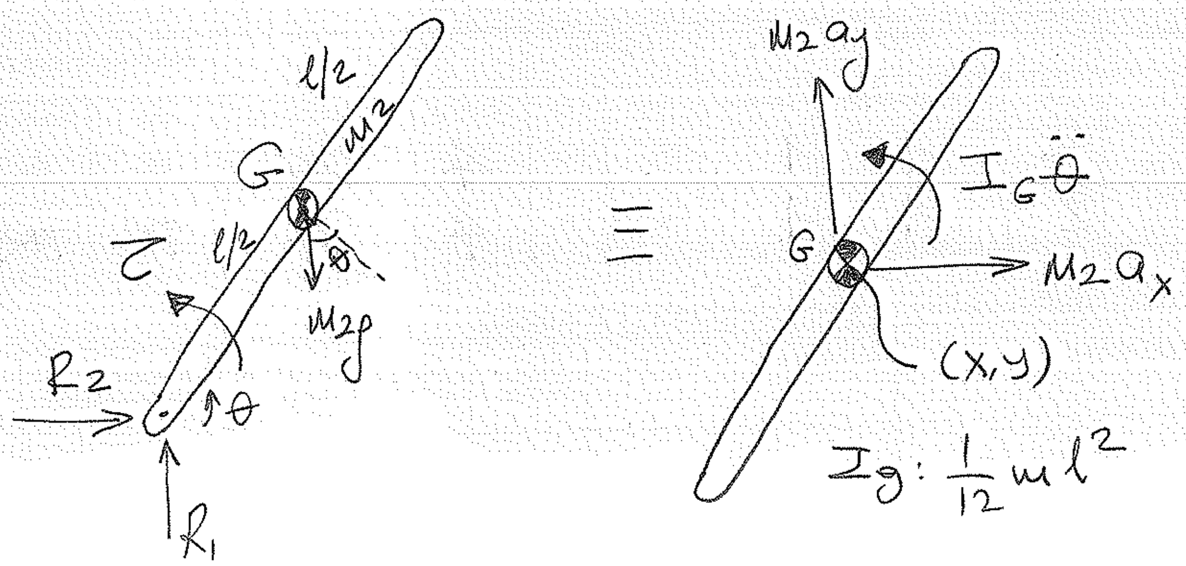
$$\Rightarrow \underline{\frac{4}{3} m l^2 \ddot{\theta} + mgl\cos\theta = 0}$$

Derive the equation of motion of the following system.



2 DoF , $q_1 = z(t)$
$q_2 = \theta(t)$

I. Newton Yöntemi



$I_g : \frac{1}{12} m l^2$

$\sum \vec{F_x} = m_2 \vec{a_x} \Rightarrow \quad R_2 = m_2 a_x$

$\sum \vec{F_y} = m_2 \vec{a_y} \Rightarrow \quad R_1 - m_2 g = m_2 . a_y$

$\sum \vec{M_G} = I_G \ddot{\theta} \Rightarrow \quad \tau - R_1 . \frac{l}{2} \cos\theta + R_2 \frac{l}{2} \sin\theta = I_G . \ddot{\theta}$

$$X = z + \frac{l}{2}\cos\theta \Rightarrow \dot{X} = \dot{z} - \frac{l}{2}\dot{\theta}\sin\theta$$

$$y = \frac{l}{2}\sin\theta \Rightarrow \dot{y} = \frac{l}{2}\dot{\theta}\cos\theta$$

$$\ddot{X} = a_x = \ddot{z} - \frac{l}{2}\ddot{\theta}\sin\theta - \frac{l}{2}\dot{\theta}^2\cos\theta$$

$$\ddot{y} = +\frac{l}{2}\ddot{\theta}\cos\theta - \frac{l}{2}\dot{\theta}^2\sin\theta = a_y$$

$$\Rightarrow R_2 = m_2\ddot{z} - \frac{m_2 l}{2}\ddot{\theta}\sin\theta - \frac{m_2 l}{2}\dot{\theta}^2\cos\theta$$

$$\Rightarrow R_1 = m_2 g + \frac{m_2 l}{2}\ddot{\theta}\cos\theta - \frac{m_2 l}{2}\dot{\theta}^2\sin\theta$$

$\Rightarrow$ Moment denklemi:

$$\tau - \frac{l}{2}\cos\theta\left\{m_2 g + \frac{m_2 l}{2}\ddot{\theta}\cos\theta - \frac{m_2 l}{2}\dot{\theta}^2\sin\theta\right\} + \frac{l}{2}\sin\theta\left\{m_2\ddot{z} - \frac{m_2 l}{2}\ddot{\theta}\sin\theta - \frac{m_2 l}{2}\dot{\theta}^2\cos\theta\right\} = I_G.\ddot{\theta}$$

$$\tau - \frac{m_2 g.l}{2}\cos\theta - \frac{m_2 l^2\ddot{\theta}\cos^2\theta}{4} + \cancel{\frac{m_2 l^2\dot{\theta}^2}{4}\sin\theta\cos\theta} + \frac{l}{2}\sin\theta\, m_2\ddot{z} - m_2\frac{l^2}{4}\sin^2\theta\,\ddot{\theta} - \cancel{\frac{m_2 l^2}{4}\dot{\theta}^2\sin\theta\cos\theta} = I_G.\ddot{\theta}$$

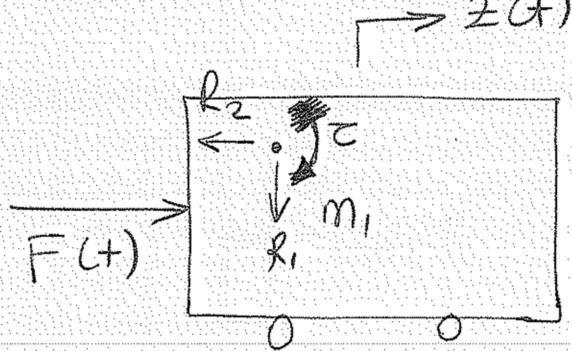
$$\Rightarrow \tau - \frac{m_2 g l}{2}\cos\theta - m_2\frac{l^2}{4}\ddot{\theta} + m_2\frac{l}{2}\sin\theta\,\ddot{z} = I_G\ddot{\theta}$$

$$\left(\frac{1}{12}m_2 l^2 + \frac{1}{4}m_2 l^2\right)\ddot{\theta} - m_2\frac{l}{2}\sin\theta\,\ddot{z} + m_2 g\frac{l}{2}\cos\theta$$

$$= \tau$$

$$\frac{1}{3}m_2 l^2\ddot{\theta} - \frac{1}{2}m_2 l\sin\theta\,\ddot{z} + m_2 g\frac{l}{2}\cos\theta = \tau$$

Birinci Hareket Denklemi.



$$\sum \vec{F_x} = m_1 . \ddot{z}$$

$$F - R_2 = m_1 . \ddot{z}$$

Not: Burada m_1 kütlesinin düşey ve rotasyonel hareketi incelenmediğinden düşey ve rotasyonel yöndeki etkiler dikkate alınmamıştır.

$$F - m_2\ddot{z} + \frac{m_2 l}{2}\ddot{\theta}\sin\theta + \frac{m_2 l}{2}\dot{\theta}^2\cos\theta = m_1\ddot{z}$$

$$\Rightarrow (m_1+m_2)\ddot{z} - \frac{m_2 . l}{2}\ddot{\theta}\sin\theta - \frac{m_2 l}{2}\dot{\theta}^2\cos\theta = F$$

İkinci Hareket denklemi..

II. Lagrange Yöntemi ile Çözüm..

$$K=\frac{1}{2}m_2(\dot{x}^2+\dot{y}^2)+\frac{1}{2}I_G\dot{\theta}^2+\frac{1}{2}m_1\dot{z}^2$$

$$K=\frac{1}{2}m_2\left\{\left(\dot{z}-\frac{l}{2}\dot{\theta}\sin\theta\right)^2+\frac{l^2}{4}\dot{\theta}^2\cos^2\theta\right\}+\frac{1}{2}\cdot\frac{1}{12}m_2l^2\dot{\theta}^2$$
$$+\frac{1}{2}m_1\dot{z}^2$$

$$K=\frac{1}{2}m_2\left\{\dot{z}^2-2\dot{z}\frac{l}{2}\dot{\theta}\sin\theta+\frac{l^2}{4}\dot{\theta}^2\sin^2\theta+\frac{l^2}{4}\dot{\theta}^2\cos^2\theta\right\}$$
$$+\frac{1}{24}m_2l^2\dot{\theta}^2+\frac{1}{2}m_1\dot{z}^2$$

$$K=\frac{1}{2}m_2\left\{\dot{z}^2-\dot{z}.l.\dot{\theta}\sin\theta+\frac{l^2}{4}\dot{\theta}^2\right\}+\frac{1}{24}m_2l^2\dot{\theta}^2+\frac{1}{2}m_1\dot{z}^2$$

$$P=m_1.g.\frac{l}{2}\sin\theta$$

$\Rightarrow$

1. $q_1=z$ için

$$\frac{\partial K}{\partial \dot{z}}=m_2\dot{z}-\frac{l\dot{\theta}\sin\theta\, m_2}{2}+m_1\dot{z}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{z}}\right)=m_2\ddot{z}-\frac{m_2 l}{2}\ddot{\theta}\sin\theta-\frac{l m_2}{2}\dot{\theta}^2\cos\theta+m_1\ddot{z}$$

$$Q_1=F$$

$$\frac{\partial K}{\partial z}=0 \quad ; \quad \frac{\partial P}{\partial z}=0$$

$\Rightarrow$ 1. Hareket denklemi : $(m_1+m_2)\ddot{z}-\frac{m_2 l}{2}\sin\theta\ddot{\theta}-\frac{l}{2}m_2\dot{\theta}^2\cos\theta=F$

2. $q_2=\theta$ için ; $Q_2=\tau$

$\cancel{\frac{\partial K}{\partial\dot{\theta}}=-m_2\dot{z}\frac{l}{2}\sin\theta+m_2\frac{l^2}{4}\dot{\theta}\sin^2\theta+m_2}$

$$\frac{\partial K}{\partial\dot{\theta}}=-\frac{1}{2}m_2\,\dot{z}.l\sin\theta+m_2\frac{l^2}{4}\dot{\theta}+\frac{1}{12}m_2 l^2\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial\dot{\theta}}\right)=-\frac{1}{2}m_2 l\,\ddot{z}\sin\theta-\frac{1}{2}m_2 l\dot{z}\dot{\theta}\cos\theta$$

$$+m_2\frac{l^2}{4}\ddot{\theta}+\frac{1}{12}m_2 l^2\ddot{\theta}$$

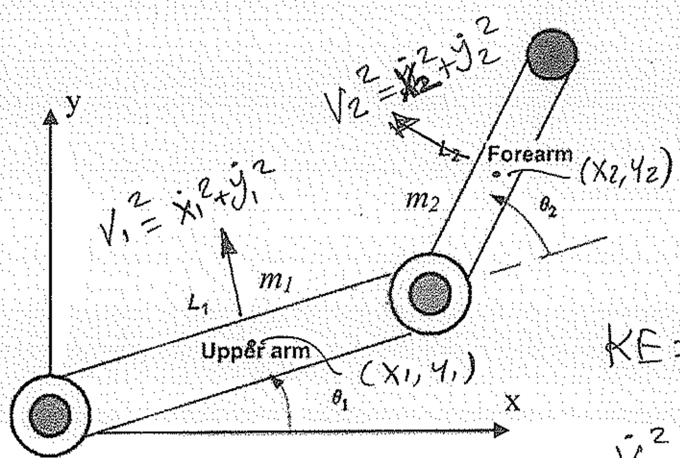
$$\frac{\partial K}{\partial\theta}=-\frac{1}{2}m_2\dot{z}\,l\dot{\theta}\cos\theta$$

$$\frac{\partial P}{\partial\theta}=m_2 g\cdot\frac{l}{2}\cos\theta$$

$\Rightarrow$ 2. Hareket Denklemi : $-\frac{1}{2}m_2 l\ddot{z}\sin\theta-\cancel{\frac{1}{2}m_2 l\dot{z}\dot{\theta}\cos\theta}+$

$$+\frac{1}{3}m_2 l^2\ddot{\theta}+\cancel{\frac{1}{2}m_2\dot{z}l\dot{\theta}\cos\theta}+m_1.g.\frac{l}{2}\cos\theta=0$$

$$\frac{1}{3}m_2 l^2\ddot{\theta}-\frac{1}{2}m_2 l\ddot{z}\sin\theta+m_2.g.\frac{l}{2}\cos\theta=0$$



Obtain the equation of motion of this biomechanical model.

Genelleştirilmiş koordinatlar:

$$q_1 = \theta_1 \quad ; \quad q_2 = \theta_2 .$$

$$KE = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} m_2 V_2^2 + \frac{1}{2} J_2 \dot{\theta}_2^2$$

$$\dot{V}_1^2 = \dot{x}_1^2 + \dot{y}_1^2 \quad ; \quad \dot{V}_2^2 = \dot{x}_2^2 + \dot{y}_2^2$$

$$\left.\begin{array}{l} x_1 = \frac{L_1}{2} . c_1 \Rightarrow \dot{x}_1 = -\dot{\theta}_1 \frac{L_1}{2} s_1 \\ y_1 = \frac{L_1}{2} . s_1 \Rightarrow \dot{y}_1 = \dot{\theta}_1 \frac{L_1}{2} c_1 \end{array}\right\} \Rightarrow v_1^2 = \frac{L_1^2}{4} \dot{\theta}_1^2$$

$$x_2 = L_1 c_1 + \frac{L_2}{2} c_{12} \Rightarrow \dot{x}_2 = -\dot{\theta}_1 L_1 s_1 - (\dot{\theta}_1 + \dot{\theta}_2) \frac{L_2}{2} s_{12}$$

$$y_2 = L_1 s_1 + \frac{L_2}{2} s_{12} \Rightarrow \dot{y}_2 = \dot{\theta}_1 L_1 . c_1 + (\dot{\theta}_1 + \dot{\theta}_2) \frac{L_2}{2} c_{12}$$

$$\Rightarrow \dot{V}_2^2 = L_1^2 \dot{\theta}_1^2 + \frac{L_2^2}{4} (\dot{\theta}_1 + \dot{\theta}_2)^2 + 2 L_1 \frac{L_2}{2} \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) c_2$$

$$I_1 = \frac{1}{12} m_1 L_1^2$$

$$I_2 = \frac{1}{12} m_2 L_2^2$$

$$\rightarrow K = \underbrace{\boxed{\frac{1}{2} m_1 \frac{L_1^2}{4}}}_{A} \dot{\theta}_1^2 + \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} m_2 V_2^2 + \frac{1}{2} J_2 \dot{\theta}_2^2$$

$$\rightarrow P = m_1 . g . \frac{L_1}{2} s_1 + m_2 . g . \left(L_1 s_1 + \frac{L_2}{2} s_{12} \right)$$

Lagrange Denklemi:

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) - \frac{\partial K}{\partial q_i} + \frac{\partial D}{\partial q_i} + \frac{\partial P}{\partial q_i} = Q_i$$

(1) $i = 1$, $q_1 = \theta_1$ için

$$\frac{\partial K}{\partial \dot{\theta}_1} = 2A\dot{\theta}_1 + J_1 \dot{\theta}_1 + \frac{1}{2} m_2 \left\{ 2 L_1^2 \dot{\theta}_1 + 2 . \frac{L_2^2}{4} (\dot{\theta}_1 + \dot{\theta}_2) + L_1 L_2 \{ \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_1 \} c_2 \right\}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_1}\right) = 2A\ddot{\theta}_1 + J_1 \ddot{\theta}_1 + \left\{ 2 L_1 \ddot{\theta}_1 + \frac{L_2^2}{2} (\ddot{\theta}_1 + \ddot{\theta}_2) + L_1 L_2 \{ 2\ddot{\theta}_1 + \ddot{\theta}_2 \} c_2 + - L_1 L_2 \{ 2\dot{\theta}_1 + \dot{\theta}_2 \} \dot{\theta}_2 s_2 \right\} * \frac{1}{2} m_2$$

$$\frac{\partial K}{\partial \theta_1} = 0 \; ; \; \frac{\partial D}{\partial \dot{\theta}_1} = 0 \; ; \; \frac{\partial P}{\partial \theta_1} = m_1 . g . \frac{L_1 . c_1}{2} + m_2 . g . L_1 . c_1 + m_2 . g \frac{L_2}{2} c_{12}$$

$\Rightarrow$ 1. Hareket Denklemi

$$2A_1 . \ddot{\theta}_1 + I_1 . \ddot{\theta}_1 + \frac{1}{2} m_2 \left(2L_1 \ddot{\theta}_1 + \frac{L_2^2}{2} (\ddot{\theta}_1 + \ddot{\theta}_2) + L_1 L_2 \{ 2\ddot{\theta}_1 + \ddot{\theta}_2 \} c_2 \right.$$

$$\left. - L_1 L_2 \{ 2\dot{\theta}_1 + \dot{\theta}_2 \} \dot{\theta}_2 s_2 \right) + m_1 g \frac{L_1}{2} c_1 + m_2 g \{ L_1 c_1 + \frac{L_2}{2} c_{12} \} = 0$$

(2) $i = 2$, $q_2 = \theta_2$ için

$$\frac{\partial K}{\partial \dot{\theta}_2} = \frac{1}{2} m_2 \left(\frac{L_2^2}{2} (\dot{\theta}_1 + \dot{\theta}_2) + L_1 L_2 \dot{\theta}_1 c_2 \right) + I_2 \dot{\theta}_2$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_2} \right) = \frac{1}{2} m_2 \left(\frac{L_2^2}{2} (\ddot{\theta}_1 + \ddot{\theta}_2) + L_1 L_2 \ddot{\theta}_1 c_2 - L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 s_2 \right) + I_2 \ddot{\theta}_2$$

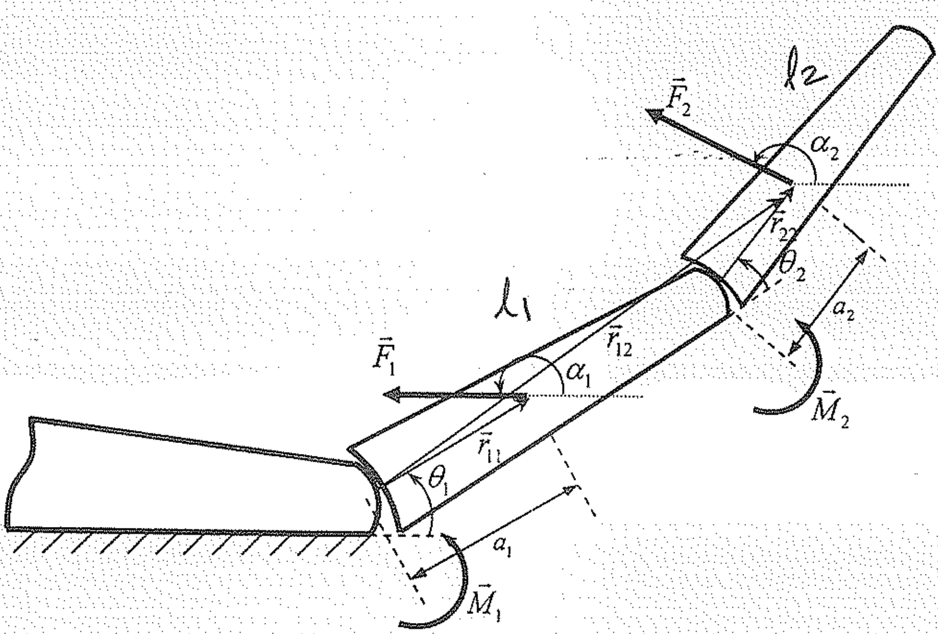
$$\frac{\partial K}{\partial \theta_2} = -\frac{m_2}{2} L_1 L_2 \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) s_2 \quad ; \quad \frac{\partial D}{\partial \dot{\theta}_2} = 0$$

$$\frac{\partial P}{\partial \theta_2} = m_2 . g \frac{L_2}{2} c_{12} \quad ;$$

$\Rightarrow$ 2. Hareket denklemi:

$$\frac{1}{2} m_2 \left(\frac{L_2^2}{2} (\ddot{\theta}_1 + \ddot{\theta}_2) + L_1 L_2 \ddot{\theta}_1 c_2 - L_1 L_2 \dot{\theta}_1 \dot{\theta}_2 s_2 \right) +$$

$$I_2 \ddot{\theta}_2 + \frac{m_2}{2} L_1 L_2 \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) s_2 + m_2 g \frac{L_2}{2} c_{12} = 0$$



Determine the joint moments M_1 and M_2 in terms of parameters of the physical system shown in the figure. Length of the first and second links are l_1 and l_2.

$$q_1 = \theta_1 \ , \quad q_2 = \theta_2$$

$$\vec{M_1} = \vec{r_{11}} \times \vec{F_1} + \vec{r_{12}} \times \vec{F_2}$$

$$\vec{M_2} = \vec{r_{22}} \times \vec{F_2}$$

$$\vec{r_{11}} = a_1 c_1 \vec{i} + a_1 s_1 \vec{j}$$

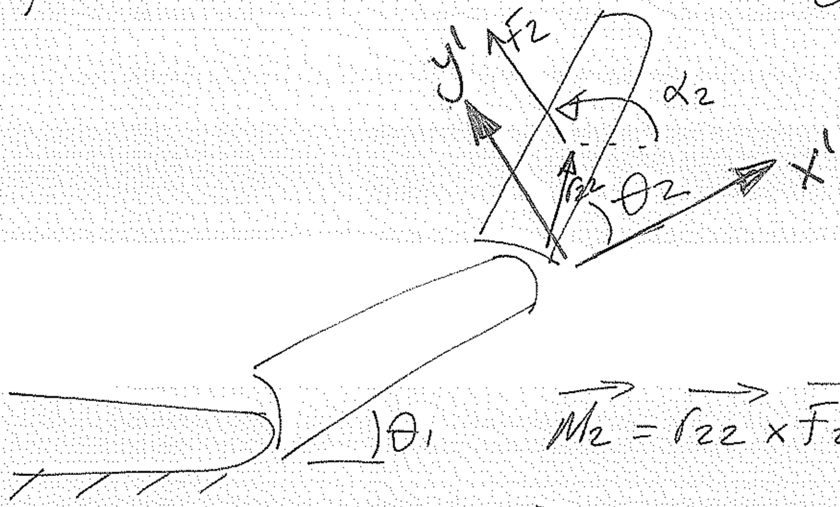
$$\vec{r_{12}} = (l_1 c_1 + a_2 c_{12}) \vec{i} + (l_1 s_1 + a_2 s_{12}) \vec{j}$$

$$\vec{r_{22}} = a_2 c_{12} \vec{i} + a_2 s_{12} \vec{j}$$

$$\vec{F_1} = F_1 . \cos\alpha_1 \vec{i} + F_2 \sin\alpha_1 \vec{j}$$

$$\vec{F_2} = F_2 \cos\alpha_2 \vec{i} + F_2 \sin\alpha_2 \vec{j}$$

Aynı $\vec{M_2}$ moment değeri, 2. eklemde konumlandırılan ~~başka~~ farklı bir koordinat ekseni ile de elde edilebilir. Bu durumda, $\vec{r_{22}}$ ve $\vec{F_2}$ bir önceki durumdan farklı olarak aşağıdaki gibi belirlenir.



$$\vec{M_2} = \vec{r_{22}} \times \vec{F_2}$$

$$\vec{r_{22}} = a_2 . c_2 \vec{i} + a_2 s_2 \vec{j}$$

$$\vec{F_2} = F_2 \cos(\alpha_2 - \theta_1) \vec{i} + F_2 \sin(\alpha_2 - \theta_1) . \vec{j}$$

__İspat__ Birinci durum için:

$$\vec{M_2} = (a_2 c_{12} \vec{i} + a_2 s_{12} \vec{j}) \times (F_2 \cos\alpha_2 \vec{i} + F_2 \sin\alpha_2 \vec{j})$$

$$\vec{M_2} = a_2 F_2 c_{12} \sin\alpha_2 \vec{k} - a_2 F_2 s_{12} \alpha_2 \vec{k}$$

$$\vec{M_2} = a_2 F_2 \sin(\alpha_2 - \theta_1 - \theta_2) \vec{k}$$

İkinci durum için.

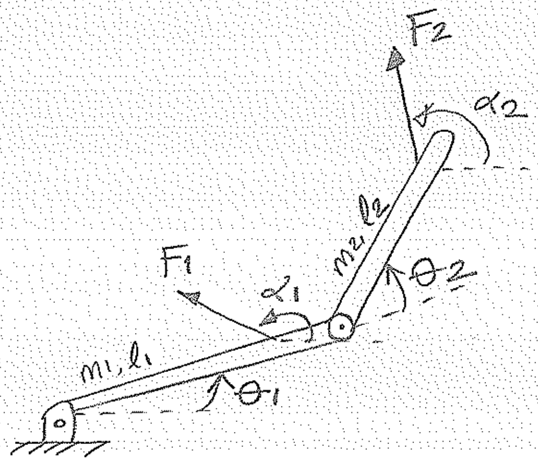
$$\vec{M_2} = (a_2 c_2 \vec{i} + a_2 s_2 \vec{j}) \times (F_2 \cos(\alpha_2 - \theta_1) \vec{i} + F_2 \sin(\alpha_2 - \theta_1) \vec{j})$$

$$\vec{M_2} = a_2 F_2 c_2 \sin(\alpha_2 - \theta_1) \vec{k} - a_2 F_2 s_2 \cos(\alpha_2 - \theta_1) \vec{k}$$

$$\vec{M_2} = a_2 F_2 \sin(\alpha_2 - \theta_1 - \theta_2) \vec{k}$$

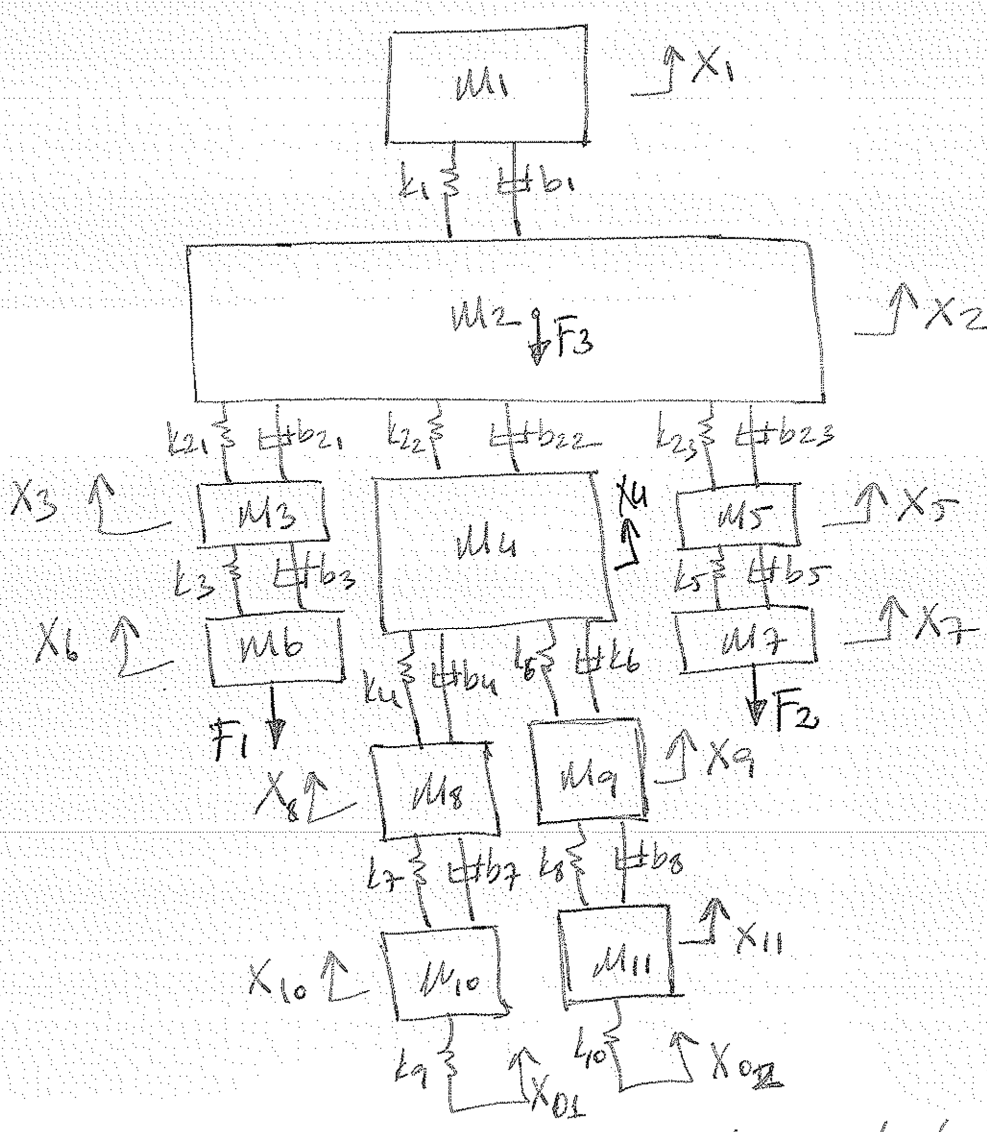
Sonuç olarak aynı $\vec{M_2}$ değeri elde edilir.

ÖRNEK



Şekildeki 2 DoF sistemin hareket denklemlerini elde ediniz.

ÖRNEK Aşağıdaki insan vücudu modelinin hareket denklemlerini elde ediniz. Yayları baştan sıkışmış kabul ediniz...



A lumped parameter model of human body.

ISIK UNIVERSITY
BIOMEDICAL ENGINEERING DEPARTMENT
MIDTERM EXAM II

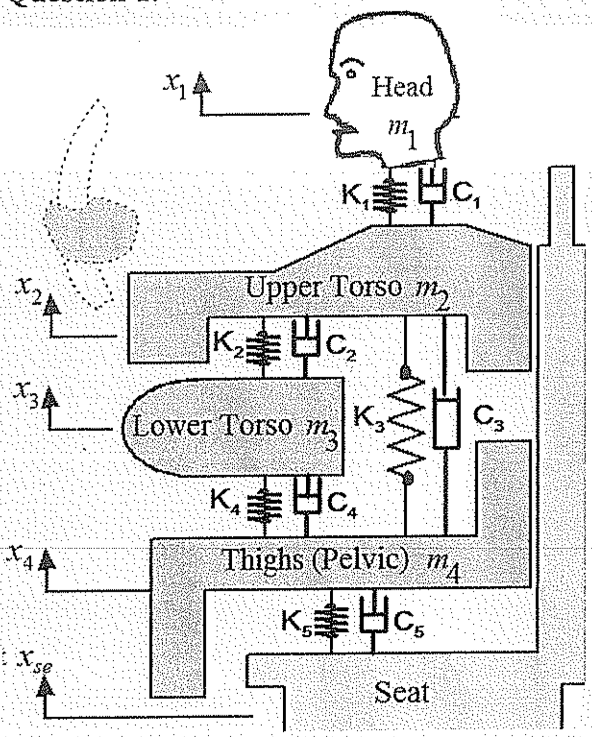
15 May 2013

Duration: 100 min.
Assist. Prof. Yunus Ziya Arslan

Name: YZA
Number:

A system with 4 DoF

Question 1.



A simplified spring–damper–mass model of the human body created to analyze the effect of vertical vehicle vibrations to human body is shown in the figure. Derive the equation of motion of the corresponding model.

$$K = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} m_3 \dot{x}_3^2 + \frac{1}{2} m_4 \dot{x}_4^2$$ (4)

$$D = \frac{1}{2} c_1 (\dot{x}_1 - \dot{x}_2)^2 + \frac{1}{2} c_2 (\dot{x}_2 - \dot{x}_3)^2 + \frac{1}{2} c_3 (\dot{x}_2 - \dot{x}_4)^2 + \frac{1}{2} c_4 (\dot{x}_3 - \dot{x}_4)^2 + \frac{1}{2} c_5 (\dot{x}_4 - \dot{x}_{sE})^2$$ (3)

$$P = \frac{1}{2} k_1 (x_1 - x_2)^2 + \frac{1}{2} k_2 (x_2 - x_3)^2 + \frac{1}{2} k_3 (x_2 - x_4)^2 + \frac{1}{2} k_4 (x_3 - x_4)^2 + \frac{1}{2} k_5 (x_4 - x_{sE})^2$$ (3)

$$\mathcal{L} : \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) - \frac{\partial K}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} + \frac{\partial P}{\partial q_i} = Q_i$$

$q_1 = x_1$

$$\frac{\partial K}{\partial \dot{x}_1} = m_1 \dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) = m_1 \ddot{x}_1$$

$$\frac{\partial D}{\partial \dot{x}_1} = c_1 (\dot{x}_1 - \dot{x}_2) \; ; \; \frac{\partial P}{\partial x_1} = k_1 (x_1 - x_2)$$

$Q_1 = 0$

Equation of Motion I

$$\Rightarrow \boxed{m_1 \ddot{x}_1 + c_1 (\dot{x}_1 - \dot{x}_2) + k_1 (x_1 - x_2) = 0}$$

(2,5)

$q_2 = x_2$

$$\frac{\partial K}{\partial \dot{x}_2} = m_2 \dot{x}_2 \Rightarrow \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \; ; \; \frac{\partial D}{\partial \dot{x}_2} = c_1 (\dot{x}_2 - \dot{x}_1) + c_2 (\dot{x}_2 - \dot{x}_3) + c_3 (\dot{x}_2 - \dot{x}_4)$$

$$\frac{\partial P}{\partial x_2} = k_1 (x_2 - x_1) + k_2 (x_2 - x_3) + k_3 (x_2 - x_4)$$

$$\Rightarrow \boxed{m_2 \ddot{x}_2 + c_1 (\dot{x}_2 - \dot{x}_1) + c_2 (\dot{x}_2 - \dot{x}_3) + c_3 (\dot{x}_2 - \dot{x}_4) + k_1 (x_2 - x_1) + k_2 (x_2 - x_3) + k_3 (x_2 - x_4) = 0}$$

Equation of motion II

$Q_2 = 0$

(2,5)

$q_3 = x_3 \quad \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}_3}\right) = m_3\ddot{x}_3 \; ; \; \frac{\partial D}{\partial \dot{x}_3} = c_2(\dot{x}_3 - \dot{x}_2) + \cancel{c_3(\dot{x}_2 - \dot{x}_4)} + c_4(\dot{x}_3 - \dot{x}_4)$

$\frac{\partial P}{\partial x_3} = k_2(x_3 - x_2) + k_4(x_3 - x_4) \quad ; \; Q_3 = 0$

$$\Rightarrow \boxed{m_3\ddot{x}_3 + c_2(\dot{x}_3 - \dot{x}_2) + c_4(\dot{x}_4 - \dot{x}_6) + k_2(x_3 - x_2) + k_4(x_3 - x_4)}$$

Equation of motion IV

2,5

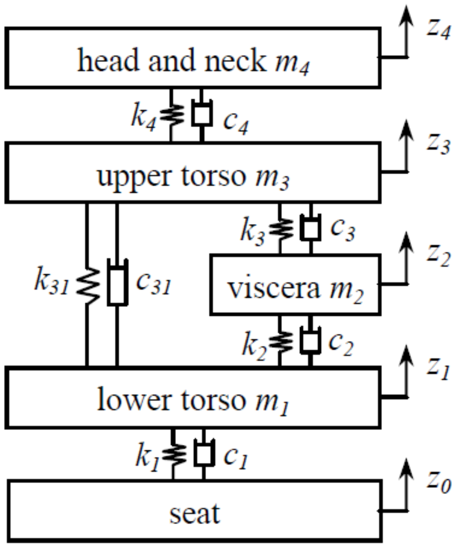
$q_4 = x_4 \quad \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}_4}\right) = m_4\ddot{x}_4 \; ; \; \frac{\partial D}{\partial \dot{x}_4} = c_3(\dot{x}_4 - \dot{x}_2) + c_4(\dot{x}_4 - \dot{x}_3) + c_5(\dot{x}_4 - \dot{x}_{5E})$

$\frac{\partial P}{\partial x_4} = k_3(x_4 - x_2) + k_4(x_4 - x_3) + k_5(x_{5E} - x_4) \quad ; \; Q_4 = 0$

$$\Rightarrow m_4\ddot{x}_4 + c_3(\dot{x}_4 - \dot{x}_2) + c_4(\dot{x}_4 - \dot{x}_3) + c_5(\dot{x}_4 - \dot{x}_{5E}) + k_3(x_4 - x_2) + k_4(x_4 - x_3) + k_4(x_{5E} - x_4) = 0$$

2,5

Question



Whole body vibration (WBV) is the transmission of low frequency environmental vibration to the human body when in contact with the vibrating surface. Long-term exposures to WBV may cause health problems such as respiratory, cardiovascular and digestive problems, reproductive organ damage, impairment of vision and balance, interference with activities and discomfort that could lead to accidents.

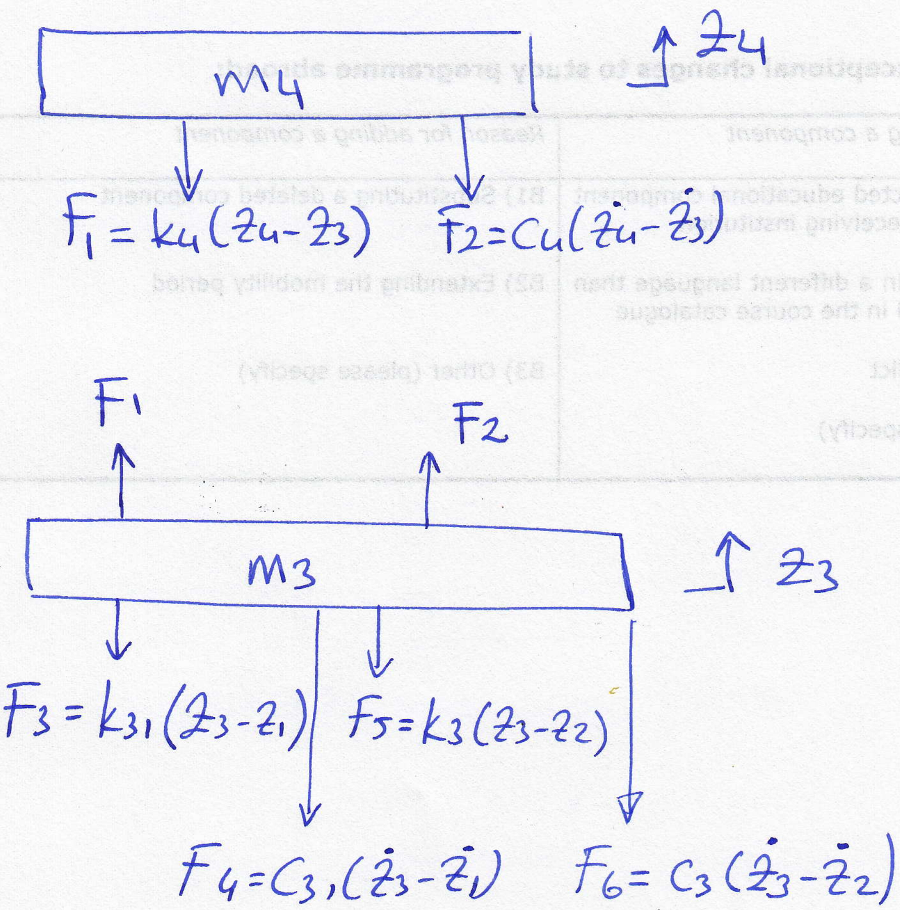
The main sources of harmful WBV in vehicles are rough road and surface conditions. To evaluate and simulate the adverse effects of WBV to human body, many biodynamic lumped-parameter human body models were proposed in the literature as seen in the figure.

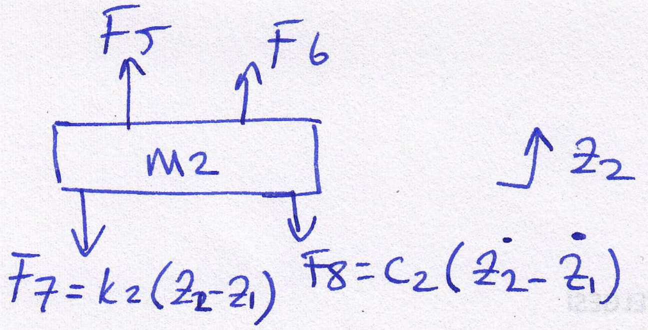
a) What is the degree of freedom of the **human body model** seen in the figure.
b) Draw the free body diagram of the **all bodies** seen in the model. Obtain the equation of motion of the whole body model using **Newtonian** approach.
c) Derive the equation of motion of the human body model, which is subjected to road disturbance z_0, using **Lagranian** approach.

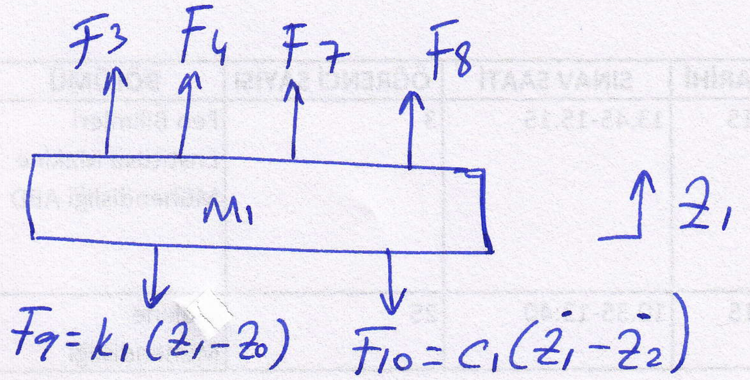
In the figures, z_0 represents the displacement of the disturbance coming from seat to human body. Displacement and mass of the body segments, coefficient of stiffness of the spring elements and coefficient of damping elements were denoted by z_i, m_i, k_i and c_i, respectively.

a, b) Free-Body Diagram and Newtonian Approach.

To draw the FBD of all body parts, first assume that all the springs are stretched at the begining. So, it is directly accepted that all springs and dashpot elements are pulling the masses and displacements of each body is higher than that of the previous one. Let's do it. DoF = 4







Tüm kütlelere dinamik denge kuralı uygulanırsa

(If the dynamic equilibrium conditions are applied to all masses, then) $\sum \vec{F} = m \cdot \vec{a}$

① $-F_1 - F_2 = m_4 \ddot{z}_4$

$$\Rightarrow \quad m_4 \ddot{z}_4 + k_4(z_4 - z_3) + c_4(\dot{z}_4 - \dot{z}_3) = 0$$

1. Equation of motion.

② $F_1 + F_2 - F_3 - F_4 - F_5 - F_6 = m_3 \ddot{z}_3$

$$m_3 \ddot{z}_3 + k_4(z_3 - z_4) + c_4(\dot{z}_3 - \dot{z}_4) + k_{31}(z_3 - z_1) + c_{31}(\dot{z}_3 - \dot{z}_1) + k_3(z_3 - z_2) + c_3(\dot{z}_3 - \dot{z}_2) = 0$$

2nd Eq. Mot.

(3) $F_5 + F_6 - F_7 - F_8 = m_2 \ddot{z}_2$

$$m_2 \ddot{z}_2 + k_3(z_2 - z_3) + c_3(\dot{z}_2 - \dot{z}_3) + k_2(z_2 - z_1) + c_2(\dot{z}_2 - \dot{z}_1) = 0$$

3rd Eq. motion.

(4) $F_3 + F_4 + F_7 + F_8 - F_9 - F_{10} = m_1 . \ddot{z}_1$

$$m_1 \ddot{z}_1 + k_{31}(z_1 - z_3) + c_{31}(\dot{z}_1 - \dot{z}_3) + k_2(z_1 - z_2) + c_2(\dot{z}_1 - \dot{z}_2)$$
$$+ k_1(z_1 - z_0) + c_1(\dot{z}_1 - \dot{z}_2) = 0.$$

Not: It is important to note that gravity effect was not included in the system. Because, it was assumed that the mechanical elements (springs + dashpots) are [illegible] in [illegible] stretched position.

c) Lagrarian Approach.

$$K=\frac{1}{2}m_1\dot{z}_1^2+\frac{1}{2}m_2\dot{z}_2^2+\frac{1}{2}m_3\dot{z}_3^2+\frac{1}{2}m_4\dot{z}_4^2$$

$$P=\frac{1}{2}k_1(z_1-z_0)^2+\frac{1}{2}k_2(z_1-z_2)^2+\frac{1}{2}k_{31}(z_1-z_3)^2$$
$$+\frac{1}{2}k_3(z_2-z_3)^2+\frac{1}{2}k_4(z_3-z_4)^2$$

$$D=\frac{1}{2}c_1(\dot{z}_1-\dot{z}_0)^2+\frac{1}{2}c_2(\dot{z}_1-\dot{z}_2)^2+\frac{1}{2}c_{31}(\dot{z}_1-\dot{z}_3)^2$$
$$+\frac{1}{2}c_3(\dot{z}_2-\dot{z}_3)^2+\frac{1}{2}c_4(\dot{z}_3-\dot{z}_4)^2$$

$\Rightarrow$ First Eq: $m_1\ddot{z}_1+c_1(\dot{z}_1-\dot{z}_0)+c_2(\dot{z}_1-\dot{z}_2)+c_{31}(\dot{z}_1-\dot{z}_3)$
$+k_1(z_1-z_0)+k_2(z_1-z_2)+k_{31}(z_1-z_3)=0$

$\Rightarrow$ Second Eq: $m_2\ddot{z}_2+c_2(\dot{z}_2-\dot{z}_1)+c_3(\dot{z}_2-\dot{z}_3)+k_2(z_2-z_1)$
$+k_3(z_2-z_3)=0$

$\Rightarrow$ Third Eq: $m_3\ddot{z}_3+k_3(z_3-z_2)+k_{31}(z_3-z_1)+k_4(z_3-z_4)$
$+c_3(\dot{z}_3-\dot{z}_2)+c_{31}(\dot{z}_3-\dot{z}_1)+c_4(\dot{z}_3-\dot{z}_4)=0$

$\Rightarrow$ Forth Eq: $m_4\ddot{z}_4+c_4(\dot{z}_4-\dot{z}_3)+k_4(z_4-z_3)=0$