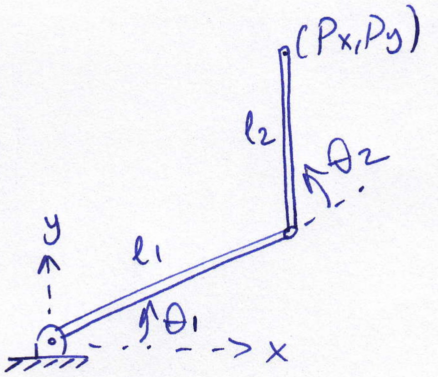
Assignment I

1. Prepare a report which indicates the fundamental aspects of
   - Stepper motor
   - AC servo motor
   - Brushless DC servo motor
   - Brushed DC servo motor

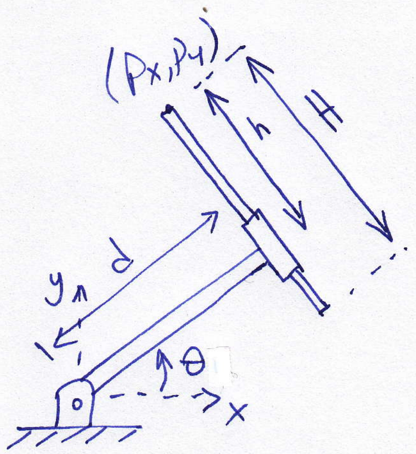
2.



Find the (P_x, P_y) point of the robot wrt θ_1 and θ_2 using Forward Kinematics.

Find the θ_1 and θ_2 of the robot wrt P_x and P_y using Inverse Kinematics.

3.

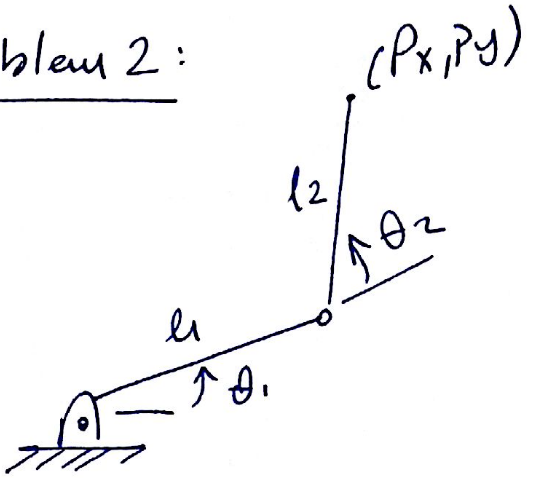


$$\left.\begin{aligned} P_x &= P_x(\theta, h) = ? \\ P_y &= P_y(\theta, h) = ? \end{aligned}\right\} \text{Forward Kinematics}$$

$$\left.\begin{aligned} \theta &= \theta(P_x, P_y) = ? \\ h &= h(P_x, P_y) = ? \end{aligned}\right\} \text{Inverse Kinematics}$$

= Solution of the Assignment I =

Problem 2:



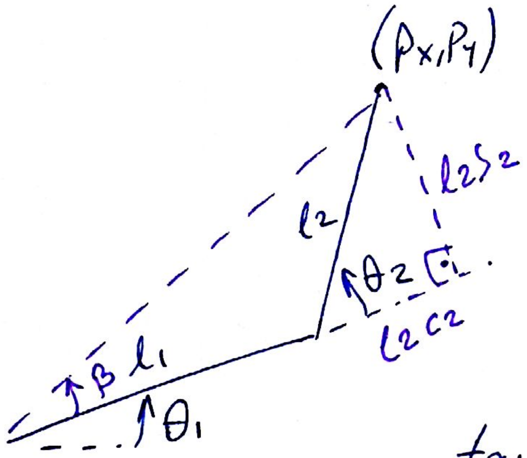
$$P_x = l_1 c_1 + l_2 c_{12}$$

$$P_y = l_1 s_1 + l_2 s_{12}$$

$$P_x^2 + P_y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 c_2$$

$$\Rightarrow c_2 = \frac{P_x^2 + P_y^2 - l_1^2 - l_2^2}{2 l_1 l_2}$$

$$s_2 = \pm\sqrt{1 - c_2^2}$$



$$\Rightarrow \theta_2 = \operatorname{Atan2}(s_2, c_2)$$

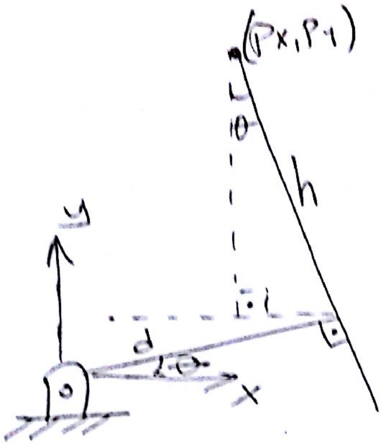
$$\tan(\theta_1 + \beta) = \frac{P_y}{P_x} \; ; \; \tan\beta = \frac{l_2 s_2}{l_1 + l_2 c_2}$$

$$\tan(\theta_1 + \beta) = \frac{\tan\theta_1 + \tan\beta}{1 - \tan\theta_1 \tan\beta} = \frac{P_y}{P_x}$$

$$\Rightarrow \cancel{\tan\theta_1} \; \tan\theta_1 = \frac{P_y - P_x . \tan\beta}{P_x + \tan\beta}$$

$$\Rightarrow \theta_1 = \operatorname{Atan2}(P_y - P_x \tan\beta, P_x + \tan\beta)$$

2. Forward and Inverse Kinematics



$$P_x = d\cos\theta - h\sin\theta$$

$$P_y = d\sin\theta + h\cos\theta$$

There is a direct mathematical representation for the Cartesian positions in terms of the link positions. (Forward Kin.)

Inverse Kin.

$$P_x^2 + P_y^2 = d^2 + h^2$$

$$h = \sqrt{P_x^2 + P_y^2 - d^2}$$

$$\left.\begin{aligned} P_x &= d\cos\theta - h\sin\theta \\ P_y &= d\sin\theta + h\cos\theta \end{aligned}\right\} \Rightarrow \quad \cos\theta = \frac{P_x \cdot d + P_y \cdot h}{d^2+h^2}, \quad \sin\theta = \frac{P_y \cdot d - P_x \cdot h}{d^2+h^2}$$

(PS! Because the inverse sine function is generally not unique and because it is sensitive to numerical variations in certain regions (specifically around $\theta = \pm 90°$), problems that also beset the inverse cosine function, neither function is directly used. Rather, the two-argument arc tangent function, Atan2 $\left(\frac{y}{x}\right)$ is employed.

Moreover, both the magnitudes and signs of X and Y are used in the definition, so that angles are uniquely defined in all four quadrants.

$$A\tan2\left(\frac{-1}{-1}\right) = 225^\circ \; ; \; A\tan\left(\frac{-1}{-1}\right) = 45^\circ$$

$$A\tan2\left(\frac{1}{1}\right) = 45^\circ \; ; \; A\tan\left(\frac{1}{1}\right) = 45^\circ$$

This distinction would be lost with the single-argument arc tangent function. Furthermore, the problems associated with division by zero (when X=0) are also avoid.

$$A\tan2\left(\frac{-2}{0}\right) = 270^\circ$$

$$\Rightarrow \theta = A\tan2\left(\frac{P_y.d - P_x.h}{P_x.d + P_y.h}\right)$$

Once desired $X_d(t)$ and $Y_d(t)$ have been specified then

$$h_d(t) = \sqrt{P_{xd}^2(t) + P_{yd}^2(t) - d^2}$$

$$\theta_d(t) = A\tan2\left(\frac{P_{yd}(t).d - P_{xd}(t)\sqrt{P_{xd}^2(t)+P_{yd}^2(t)-d^2}}{P_{xd}(t).d + P_{yd}(t).\sqrt{P_{xd}^2(t)+P_{yd}^2(t)-d^2}}\right)$$