Assignment 2- Solutions

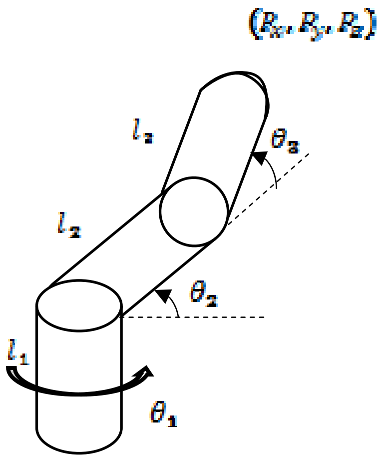
1. Aşağıda verilen rotasyon matrislerini parametrik olarak elde ediniz. (Derive the following rotation matrices as a function of θ)
   - $R_x(\theta)$ =?
   - $R_y(\theta)$ =?
   - $R_z(\theta)$ =?

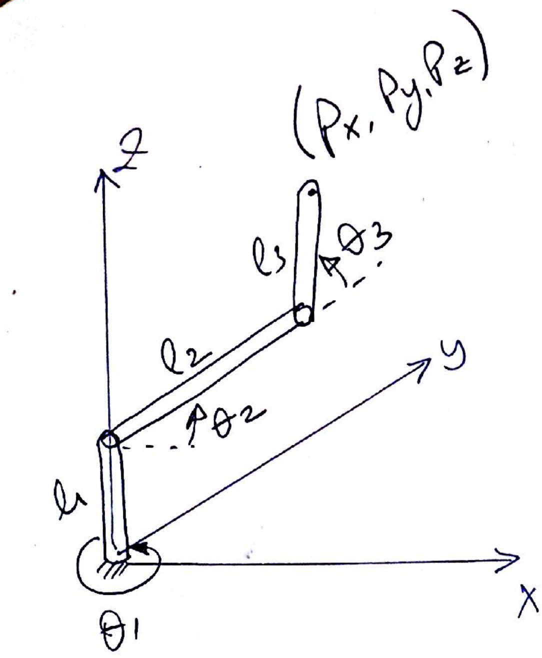
$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

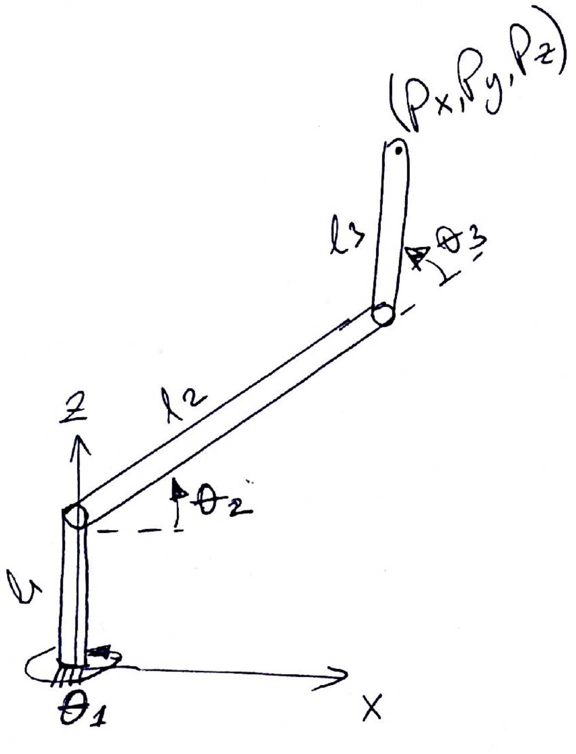
$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Şekildeki 3 serbestlik dereceli robot kolunun düz ve ters kinematik analizini yapınız (Perform the forward and inverse dynamic analysis of the following robot with 3 DoF).







<u>Forward Kinematic Solution</u>

$$P_x = (l_2 C_2 + l_3 C_{23}).C_1$$

$$P_y = (l_2 C_2 + l_3 C_{23}).S_1$$

$$P_z = l_1 + l_2 S_2 + l_3 S_{23}$$

$$\begin{matrix} P_x \\ P_y \\ P_z \end{matrix} \xrightarrow[\text{Kinematic}]{\text{Inverse}} \begin{matrix} \theta_1 = ? \\ \theta_2 = ? \\ \theta_3 = ? \end{matrix}$$

$$P_x = (l_2 C_2 + l_3 C_{23}) \cdot C_1$$

$$P_y = (l_2 C_2 + l_2 C_{23}) \cdot S_1 \quad \Rightarrow \quad \frac{P_x}{P_y} = \frac{C_1}{S_1}$$

$$\Rightarrow \tan\theta_1 = \frac{P_y}{P_x} \Rightarrow \theta_1 = A\tan 2\,(P_y, P_x)$$

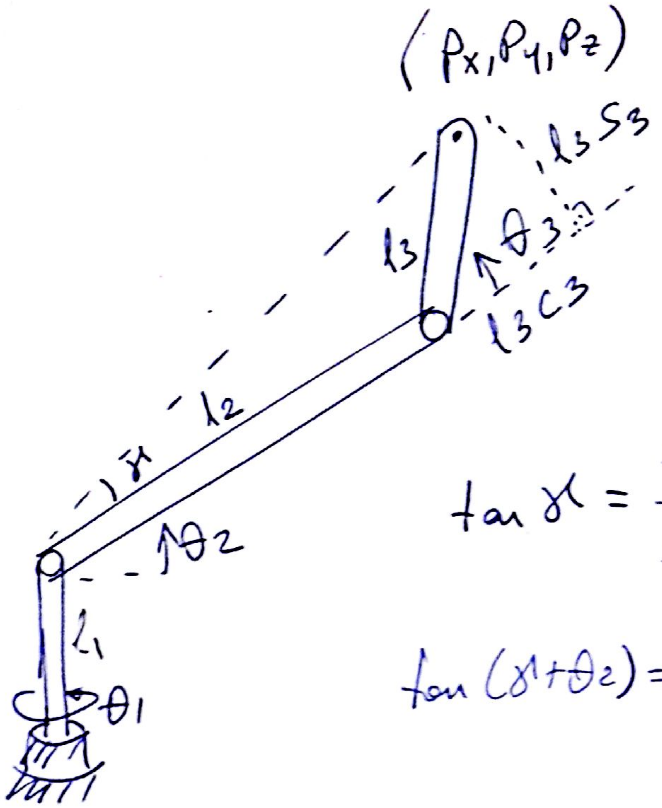
$$\left.\begin{aligned} \frac{P_x}{C_1} &= l_2 C_2 + l_3 C_{23} \\ P_z - l_1 &= l_2 S_2 + l_3 S_{23} \end{aligned}\right\} \Rightarrow$$

$$\frac{P_x^2}{C_1^2} + (P_z - l_1)^2 = l_2^2 C_2^2 + l_3^2 C_{23}^2 + 2 l_2 l_3 C_2 C_{23} + l_2^2 S_2^2 + l_3^2 S_{23}^2 + 2 l_2 l_3 S_2 S_{23}$$

$$\frac{P_x^2}{C_1^2} + (P_z - l_1)^2 = l_2^2 + l_3^2 + 2 l_2 l_3 C_3$$

$$\Rightarrow \cos\theta_3 = \boxed{\frac{\frac{P_x^2}{C_1^2} + (P_z - l_1)^2 - l_2^2 - l_3^2}{2 l_2 l_3}}\; M$$

$$\sin\theta_3 = \pm\sqrt{1 - M^2} \Rightarrow \theta_3 = A\tan 2\,(\sin\theta_3, \cos\theta_3)$$



$$\tan\gamma = \frac{l_3 S_3}{l_2 + l_3 C_3} \; ; \; \tan(\gamma + \theta_2) = \frac{\tan\theta_2 + \tan\gamma}{1 - \tan\theta \tan\gamma}$$

$$\tan(\gamma + \theta_2) = \frac{P_z - l_1}{\sqrt{P_x^2 + P_y^2}}$$

$$\underbrace{\frac{P_z - l_1}{\sqrt{P_x^2 + P_y^2}}}_{N} = \frac{\tan\theta_2 + \tan\gamma}{1 - \tan\theta_2 \tan\gamma} \Rightarrow N - \tan\theta_2 \tan\gamma = \tan\theta_2 + \tan\gamma$$

$$\tan\theta_2 (1 + \tan\gamma) = N - \tan\gamma$$

$$\tan\theta_2 = \frac{N - \tan\gamma}{1 + \tan\gamma}$$

$$\Rightarrow \theta_2 = \mathrm{Arctan2}(N - \tan\gamma, 1 + \tan\gamma)$$

$$\theta_2 = \mathrm{Arctan2}\left(\frac{P_z - l_1}{\sqrt{P_x^2 + P_y^2}} - \frac{l_3 S_3}{l_2 + l_3 C_3}, 1 + \frac{l_3 S_3}{l_2 + l_3 C_3}\right)$$

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